Balanced Assessment

MIDDLE GRADES
ASSESSMENT

PACKAGE 1

Balanced Assessment for the Mathematics Curriculum

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Expanded Table of Contents *

Long Tasks	Task Type	Circumstances of Performance
1. Border Tiles	45-minute pure investigation; open-ended; nonroutine context, loosely related to adult life	individual assessment after a whole class introduction and some entry work in pairs
2. Table Tennis	45-minute nonroutine problem; applied power in a student-life context; nonroutine in both context and mathematical content	individual assessment after a whole class introduction
3. Consecutive Addends	60-minute pure investigation; open-ended nonroutine context	individual assessment after a whole class introduction
4. Emergency 911! Bay City	60-minute open-ended evaluation and recommendation problem; applied power in an adult-life, nonroutine context	pair assessment
5. Sum of Seven	30-minute evaluation problem; applied power in a student-life context, currently nonroutine	individual assessment
6. T-shirt Design	30-minute re-presentation of information; nonroutine open-ended problem; illustrative application	individual assessment after a whole class introduction
7. Energy	60-minute re-presentation of information; nonroutine open-ended problem; applied power in an adult-life context	individual assessment after a whole class introduction

* For explanations of terms that may be unfamiliar, see the Glossary, and the *Dimensions of Balance* table in the Introduction

Mathematical Content	Mathematical Processes
Patterns and functions in a pure geometric context	formulation of relationships through conjecture, verification, and generalization; some manipulation
Other mathematics: a focus on combinatorics with some practical number work	formulation and manipulation are evenly balanced
Patterns and generalizations in a pure number context	formulation and modeling are used with some drawing of conclusions
Representing and interpreting data and statistics	inferring from given data and drawing conclusions with a written report
Finding and interpreting probabilities, either experimental or theoretical	modeling of a probability experiment
Locating and describing geometric shapes and space	modeling a way to describe a design and completing a written report
Data and statistics; designing a graph or visual representation of the data	interpretation of data and transformation from one representation to another

Expanded Table of Contents

Short Tasks	Task Type	Circumstances of Performance
8. Best Guess?	15-minute evaluation task; applied power; nonroutine context and argument	individual assessment
9. Fractions of a Square	25-minute collection of four short tasks; an exercise in pure mathematics; some nonroutine open-ended questions	individual assessment
10. Similar Triangles	10-minute exercise in pure mathematics; a collection of four smaller tasks	individual assessment
11. Disk Sum Problem	25-minute problem in pure mathematics; nonroutine context	individual assessment
12. Stacking Cubes	15-minute exercise to illustrate mathematical ideas	individual assessment
13. Drop and Bounce	20-minute re-presentation task, broken into parts up to 5 minutes long; illustrative application of the exponential in nonroutine context	individual assessment

Extended Task	Task Type	Circumstances of Performance
14. Parachutes	180-minute rich design task; applied power in a student-life context; open-ended and nonroutine in context, mathematical results and connections	small-group assessment after a whole-class introduction and group experimental work

Mathematical Content	Mathematical Processes
Handling data: a statistical analysis of 3 sets of 5 numbers, where spread is the dominant effect; associated number work	balances formulation, manipulation, and interpretation beyond the mechanical application of rules.
Number and quantity; working with rational numbers in a geometric context	manipulation of shapes and rational numbers, some checking and evaluation of work is required
Geometry, space, and shape; working with proportions and scaling in a geometric context; some generalization	mainly checking and evaluation
Number and quantity; adding decimals and producing solution sets that satisfy given constraints	formulation, manipulation, and checking of results
Geometry, space, and shape; using 3-D spatial reasoning and determining surface areas and volumes	representation of spatial information, modeling and formulation followed by manipulation
Algebra and functions; translation to numerical, graphical, and symbolic-algebra form of a verbal description of exponential behavior	mainly manipulation, with some formulation in the early parts

Mathematical Content	Mathematical Processes
Data and statistics link to the game's geometry in the design and evaluation of a game; simulation, collection of experimental data, and probability inferences for the shapes involved	a balance between formulation, manipulation, and interpretation, with evaluation of results

Task 1

Border Tiles

Overview

Investigate conjectures.
Search for patterns and generate rules.
Draw conclusions about the generalizability of rules.

Long Task

Task Description

This task asks students to investigate the use of different types of tiles to create rectangles with borders around their perimeter. Students form conjectures about patterns and rules for the number of tiles needed to make different classes of rectangles.

Assumed Mathematical Background

It is assumed that students have had experiences with open-ended problems that engage them in investigating a situation, making conjectures, searching for patterns, and drawing conclusions from their findings.

Core Elements of Performance

- identify classes of rectangles relevant to the investigation
- pose conjectures about rules that relate the number of tile-types needed to make different classes of rectangles
- systematically investigate the conjectures
- draw appropriate conclusions about the generalizability of the conjectures

Circumstances

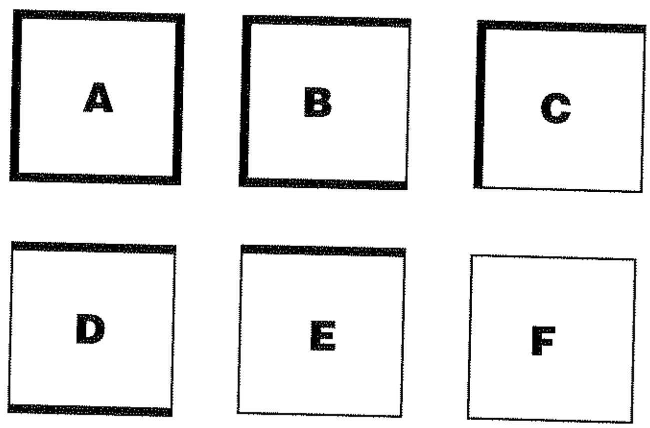
Grouping:	Following a class introduction, students do some entry work in pairs, and then complete an individual written response.
Materials:	dot paper, graph paper, and models of tiles cut from transparency
Estimated time:	45 minutes

Name Date

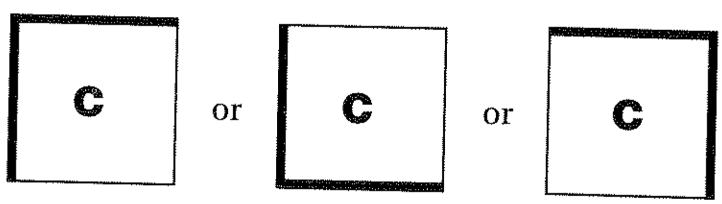
Border Tiles

As a class

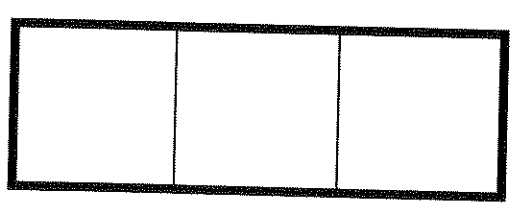
When tiling walls and floors, sometimes people use special tiles that have borders on them. Square tiles can have borders on any number of edges. The six possible border tiles are shown below.



Each of these tiles can be flipped, or rotated, and still keep the same letter name. For example, the C-tile can look like any of the tiles shown here.



Some of these tiles can be placed together to make rectangles with dark borders around the perimeter. For example:



Name Date

1. Luke calls the rectangle on the bottom of the previous page a 1-by-3 rectangle since it is one square tile wide and three square tiles long. How many of each type of tile are in this 1-by-3 rectangle? ______________________________

2. How many of each type of tile would be in a 1-by-6 rectangle? ______

3. Luke noticed similarities in all of the rectangles that are one square tile wide. He created a class of rectangles he called "1-bys." For any rectangle in Luke's class of "1-bys," find how many of each type of tile would be needed. ______________________________

____

With a partner

Create a class of rectangles different from Luke's "1-bys." Look for a general rule that gives the number of each type of tile used in your class of rectangles.

Name

Date

This problem gives you the chance to

- *formulate conjectures*
- *systematically investigate conjectures*
- *find patterns and rules*
- *justify why the patterns and rules generalize across cases*

On your own

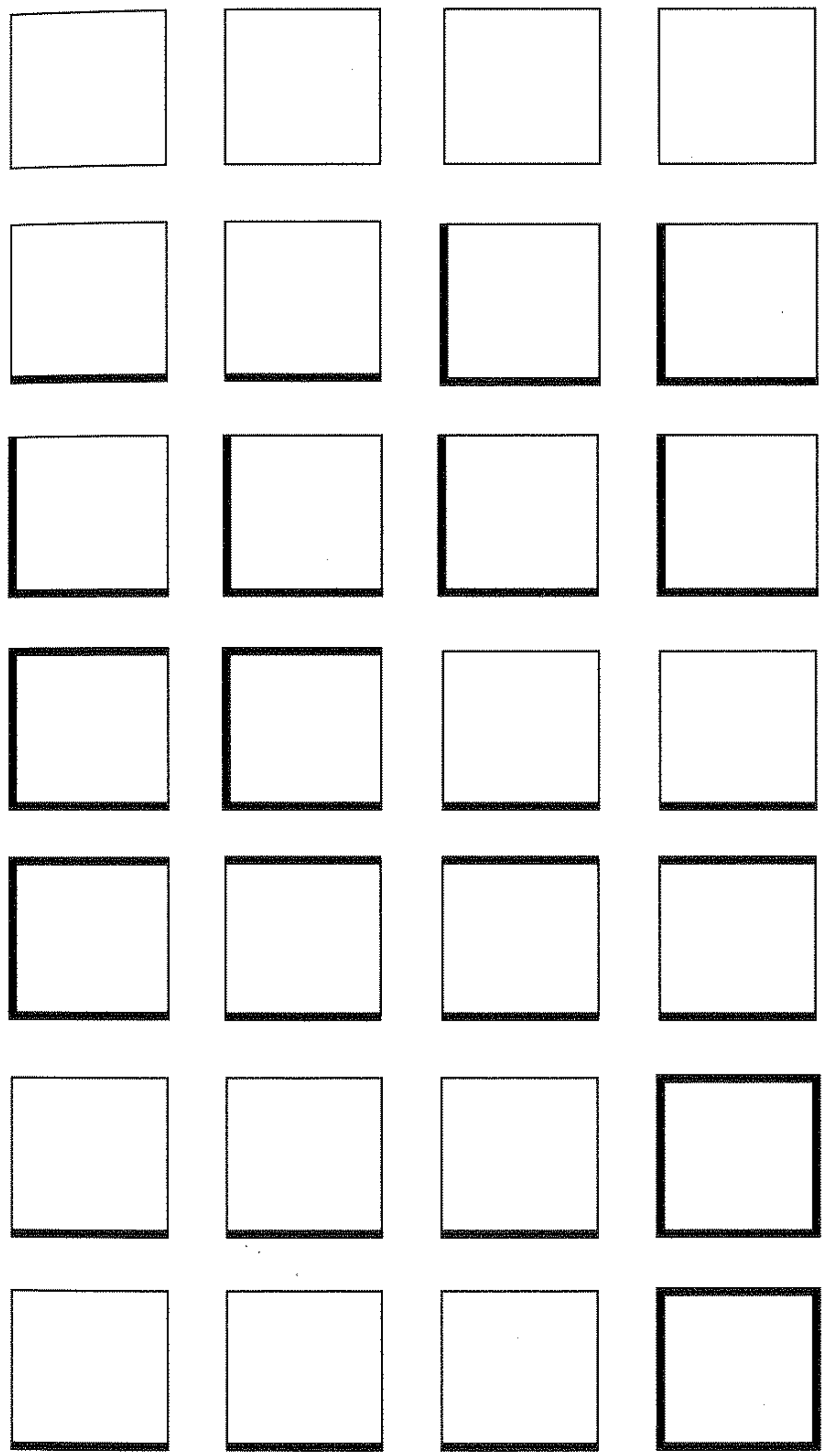
Write a general rule for the number of each type of tile in a class of rectangles.

Justify or explain why your rules work for all the rectangles in the class.

Do this for as many classes of rectangles as you can.

Name Date

Border Tiles Transparency



Name Date

Name Date

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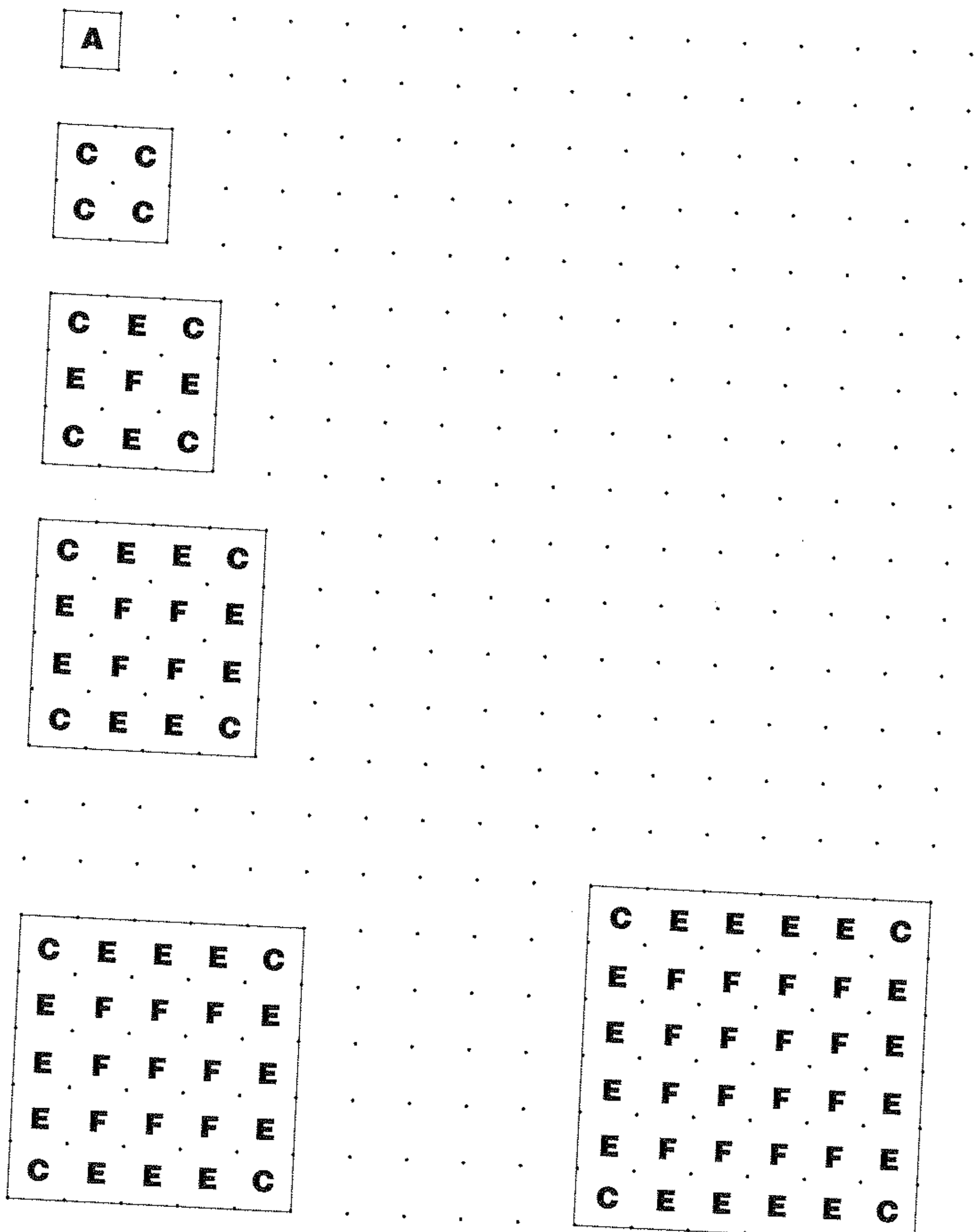
Task 1

A Sample Solution

The following is an example of a drawing a student might do to investigate square rectangles.



The following table summarizes the results found by making squares.

	A	B	C	D	E	F
1-by-1	1	0	0	0	0	0
2-by-2	0	0	4	0	0	0
3-by-3	0	0	4	0	4	1
4-by-4	0	0	4	0	8	4
5-by-5	0	0	4	0	12	9
6-by-6	0	0	4	0	16	16
***n*-by-*n* (for $n \geq 2$)**	0	0	4	0	$4(n-2)$	$(n-2)^2$

The following are some rules about the numbers of different types of tiles needed to make the class of rectangles that are squares.

A-tile: An A-tile makes a 1-by-1 square by itself. A-tiles are not used in any other squares.

B-tile: There are no B-tiles used in any squares since three of its sides are borders and adding another tile to complete the border would no longer make a square.

C-tile: These tiles are used to make the 4 corners of any rectangle that is not a "1-by," including squares that are at least 2-by-2.

D-tile: There are no D-tiles used in squares. D-tiles are only used in "1-bys."

E-tile: E-tiles are used as noncorner edge tiles of any rectangle that is not a "1-by," including squares. Starting with 4 E-tiles in a 3-by-3 square, the number of E-tiles increases to 8 tiles in a 4-by-4 square, 12 tiles in a 5-by-5 square, 16 tiles in a 6-by-6 square, and so on. As you increase the length of the square by one tile, 4 E-tiles are added to the figure—one to each side. For any n-by-n square with n greater than or equal to 2, there are $4(n-2)$ E-tiles.

F-tile: To make a 3-by-3 square, 1 F-tile is needed. Four F-tiles are needed in a 4-by-4 square, 9 F-tiles in a 5-by-5 square, 16 F-tiles in a 6-by-6 square, and so on. Since F-tiles are the "inside" pieces, they form a square inside the original square. For any n-by-n square with n greater than or equal to 2, there are $(n-2)^2$ F-tiles.

Task 1

The following are some results found when investigating the class of 2-by-n rectangles.

A-tile: No 2-by-n rectangle can contain an A-tile; an A-tile makes a square by itself because all of its edges are borders.

B-tile: The 2-by-n B-tiles can only be used in a 2-by-1 rectangle. This rectangle uses exactly two B-tiles.

C-tile: Since the C-tile is a corner tile, all 2-by-n rectangles, where n is greater than 1, contain exactly 4 C-tiles.

D-tile: Since D-tiles can only be used in the class of 1-by-n rectangles of $n > 2$ (because the opposite edges are borders), there will be no D-tiles in 2-by-n rectangles.

E-tile: The number of E-tiles in a 2-by-n rectangle, where n is greater than 1, will always be $2(n - 2)$. Each row has n tiles, 2 of these tiles are C-tiles, and the others are E-tiles because only one edge is a border. Since there are two rows of C-tiles and E-tiles, the total number of E-tiles will be $2(n - 2)$.

F-tile: The 2-by-n class of rectangles will not have any F-tiles. Because F-tiles do not have a border, they are used "inside" rectangles and thus are only used in m-by-n rectangles where m and n are both greater than or equal to 3.

Students' investigations can include multiple dimensions of rectangles. For a correct solution, students need to identify a class of rectangles to consider (for example, squares, 2-bys), and then investigate how many of the different types of tiles are used in the class. Based on their investigations, students must describe the patterns they find and pose rules. Students' work should show evidence of conducting systematic investigations such as testing a variety of cases for a given class of rectangles, searching for a counterexample, and making deductive arguments. Finally, students should draw appropriate conclusions about their rules based on the investigations they have conducted. For example, if students find one counterexample, they may conclude that the rule does not hold. They may even go on to identify under what conditions the rule holds and under what conditions it does not hold.

More on the Mathematics

The following solution summarizes the rules for all n by m rectangles:

$n = m = 1$	1 A-tile
$n = 1, m > 1$	2 B-tiles, $(m - 2)$ D-tiles
$n > 1, m > 1$	4 C-tiles, $[2(n + m) - 8]$ E-tiles, $[(n - 2)(m - 2)]$ F-tiles

Using this Task

Task 1

Before administering this assessment you may want to make and cut out the border tiles from the sheet included (see page 5). Dot and graph paper are included in the activity and should be available for students to use.

To launch the task, distribute the first two pages of the task to students. Read aloud the top of the first page and address the first question on the second page as a class. Have students label each piece in the given rectangle to help them understand that orientation does not make a tile different (this drawing contains only B-tiles and a D-tile). Students should work in pairs on questions 2 and 3 for no more than 3-4 minutes. Have some of the students share their responses to these questions with the class to make certain that all students understand how the tiles are used.

Continue the launch by reading with the class the instructions for *With a partner* and then give students no more than 15 minutes to work in pairs to brainstorm patterns or rules to investigate. Make sure students understand what is meant by a "class of rectangles." You may want to reiterate that Luke found a class of rectangles that he called "1-bys."

After students have explored the problem with a partner, distribute the individual assessment sheet entitled *On your own*. Read aloud the aims of the assessment found in the box on the top of page 4. Students should then work individually for at least 20 minutes. The individual work will be assessed.

Issues for Classroom Use

Posing and investigating conjectures and searching for ways to verify whether and under what conditions a conjecture holds is part of the essence of mathematics. Students who have had little experience with open-ended investigations may have difficulty starting this task and sustaining effort.

Students may have difficulty determining whether or not a pattern or rule can be generalized to all cases. Frequently, students test strategically selected cases and then claim with certainty that the pattern or rule holds true for *all* cases. In fact, given the extent of their investigations students can claim that the conjecture seems to be valid, but may not necessarily be true for all cases. A generalization for cases can be made either by testing every case (in a finite set of cases), by making a deductive argument (using natural and/or symbolic language), or by finding a counterexample (in which case the rule or pattern does not hold).

Task 1

Characterizing Performance

This section offers a characterization of student responses and provides indications of the ways in which the students were successful or unsuccessful in engaging with and completing the task. The descriptions are keyed to the *Core Elements of Performance.* Our global descriptions of student work range from "The student needs significant instruction" to "The student's work meets the essential demands of the task." Samples of student work that exemplify these descriptions of performance are included below, accompanied by commentary on central aspects of each student's response. These sample responses are *representative;* they may not mirror the global description of performance in all respects, being weaker in some and stronger in others.

The characterization of student responses for this task is based on these *Core Elements of Performance:*

1. Identify classes of rectangles relevant to the investigation.
2. Pose conjectures about rules that relate the number of tile-types needed to make different classes of rectangles.
3. Systematically investigate the conjectures.
4. Draw appropriate conclusions about the generalizability of the conjectures.

Descriptions of Student Work

The student needs significant instruction.

Student shows evidence of a search but states no rule(s), states incorrect/incomplete rule(s), or states only rule(s) for "1-bys."

Student A

Student A shows evidence of searching for a pattern, but only for the "1-by-n" rectangles that were investigated in the pre-assessment activity. The rule posed by Student A is incomplete "2 B's + # D's = # tiles." Although implicit in the rule is that every "1-by-x" rectangle has 2 B-tiles in it, the rule doesn't provide a way to generate the number of D-tiles given information about the dimensions of a "1-by" rectangle.

The student needs some instruction.

Student poses at least two plausible rules for cases beyond the "1-bys." There is an attempt to justify the rules, but the support is nonexistent or weak (for example, the student does not test an adequate number or variety of cases for the conclusion drawn; student does not delineate the rectangles for which the rules hold).

Student B

Student B considers all six tiles. However, only the rules for tiles B and C implicitly address the number of tiles used for a class of rectangles, as requested in the directions. The rules for D-, E-, and F-tiles address when the tiles are used and when they are not. The class of rectangles for the A-tiles and C-tiles refer to classes different from the "1-bys," but the student provides weak support for each of the rules: only one example of each is considered on the dot paper. Student B correctly rules out appropriate cases for tiles A, C, D, E, and F, but for tile B, the student neglects to mention that the rule does not hold for 1-by-1 rectangles. To have made a score at the next higher level, the student would need to provide strong support for at least two of the rules.

The student's work needs to be revised.

Student poses at least two complete and correct rules about the number of tiles needed for classes of rectangles, and at least one rule is for a class beyond the "1-bys." (There may also be some incomplete or incorrect rules.) The student also provides *strong* support for the plausibility of at least two of the rules: he or she tests an adequate number and variety of cases and delineates the cases for which the rules hold. (There may be some errors and omissions.)

Student C

Student C makes rules about five tiles: B, D, C, E, and F. The rules for tile B and tile D are clear and correct for the "1-by" class of rectangles, and *generalized* reasoning is used to support the rule for the B- and D-tiles. The rule for C-tiles is clear and correct for both the "2-by" and "3-by" rectangles, and strong support for the C-tiles rule is made in the table for the "3-by" class of rectangles. Although the rules for tiles E and F do not make it possible to generate the number of tiles given the dimensions of the rectangle, the patterns of increase are correctly related to incremental increases in one dimension of the "3-by" rectangle. Although the rules for the E-tile and F-tile are incomplete, they are plausible rules, and the student provides support for the plausibility of the E and F rules in the table of "3-by" test cases. Finally, Student C clearly describes the cases for which the B-, D-, and C-tiles' rules hold, by ruling out the appropriate cases.

Task 1

The student's work meets the essential demands of the task.

Student poses at least two complete and correct rules about the number of tiles needed for a class of rectangles beyond the "1-bys." The student also provides *generalizable* support for at least two of the rules: explains why the rules hold for all rectangles in the class, and delineates the cases for which the rules hold.

Student D

Student D investigates the class of square and poses correct and complete rules for tiles C, E, and F. The explanations given are *generalizable* to all members of the class.

Student A

#3

#B	1 By X
2	1 By 2
2	1 By 3
2	1 By 4
2	1 By 5
2	1 By 6
2	1 By 7

#D	1 By X
0	2
1	3
2	4
3	5
4	6
5	7

So 2 B's + #D's = # tiles
tiles is = to x in 1 By X
Since It's One times the tiles
in length

Student B

A tiles: these tiles are only used in a 1 by 1 rectangle

B tiles: these are only used in 1 by — rectangles and are on opposite ends of the rectangles

C tiles: these tiles are used as corners and only occur in a 2 by 2 or larger rectangle.

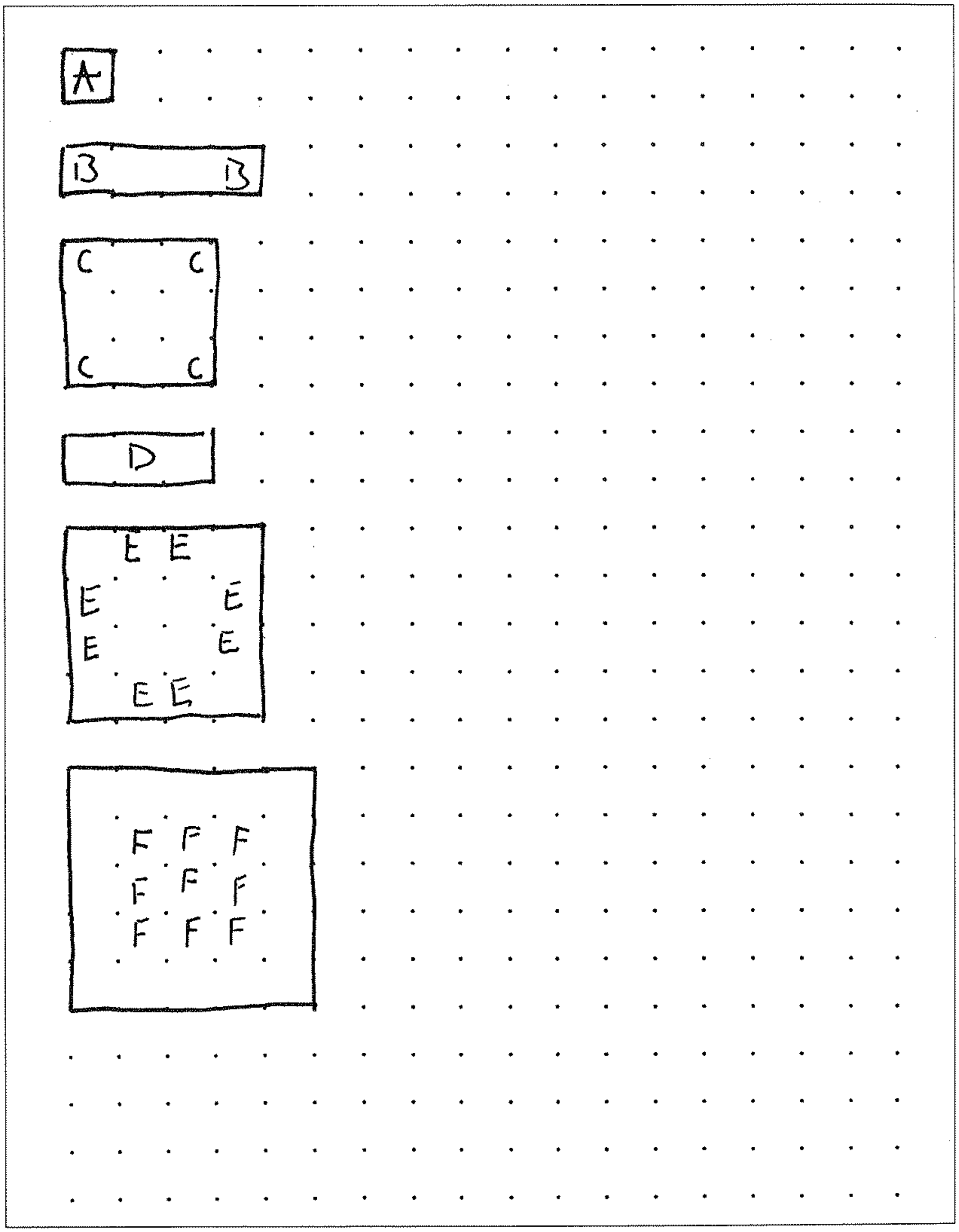
D tiles: these tiles are used in a 1 by situation but are never found in 1 by 1 rectangles.

E tiles: these tiles are used in rectangles equal to or larger than 2 by 3 rectangles.

F tiles: these tiles have no border and are sanwiched between other tiles it only occurs in rectangles 3 by 3 or larger.

See Grid for Examples

Student B



Student C

Rule for 1-by

For every "1-by", except for 1X1, there are 2 b tiles. All the tiles in between the 2 b's are d's. When you add the b's to the d's you should get the ~~#~~ of files in the rectangle.

Rule for 2-by

For every "2-by", except for 2x1, there are 4 c's. As each 2-by increases, the e tiles increase by 2, starting at zero.

Rule for 3-by

The rule for 3-by's is exactly the same as the rule for the 2-by's, but starting with 2. In 3-by's.

Rule for 4-by

Same as 3-by rule.

Student C

3 bys	a	b	c	d	e	f
3X1		2		1		
3X2			4	[illegible]	2	
3X3			4		4	1
3X4			4		6	2
3X5			4		8	3
3X6			4		10	4
3X7			4		12	5
3X8			4		14	6
3X9			4		16	7
3X10			4		18	8
3X11			4		20	9
3X12			4		22	10

For every "3-by", except for 3X1, there are 4 c's. As each 3-by increases, the e tiles increase by 2. From 3x3 on there are f tiles too that increase by 1 for every increase of the 3-by's.

Student D

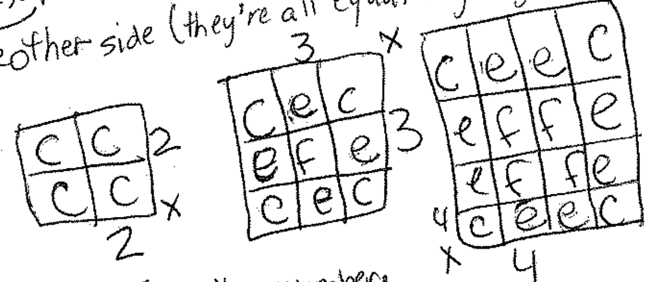
Squares:

Table A: Squares

Sides	C	E	F	TOTAL
2X2	4	0	0	4
3×3	4	4	1	9
4X4	4	8	4	16
5X5	4	12	9	25
6X6	4	16	16	36
7X7	4	20	25	49
8X8	4	24	36	64
9X9	4	28	49	81
10X10	4	32	64	100

Connections: A is only used in the first square so I didnt put it in the table. C always remains as 4 because no matter how the Perimeter changes, a square always has 4 corners. E changes by 4 every trial because if one side changes by one, then the total change is 4 (4 sides to a square). F is the length of one side (minus the corners) times the length of the other side (they're all equal any ways!) - minus the corners.

EXAMPLES →



So if you take 2 away from the numbers of tiles on a side E is 4 times that.

Task 2

Overview

Design and apply a method for counting.

Table Tennis

Long Task

Task Description

This task asks students to determine how many matches and how much time are needed to run a round-robin table tennis tournament, in which each player is matched in turn against every other player.

Assumed Mathematical Background

It is assumed that students have had experience conducting systematic investigations.

Core Elements of Performance

- design a method for determining the number of games in a round-robin tournament
- analyze and convert among units of measure to determine the amount of time needed for a tournament

Circumstances

Grouping:	Following a class introduction, students complete an individual written response.
Materials:	No special materials are needed for this task.
Estimated time:	45 minutes