

ADVANCED HIGH SCHOOL ASSESSMENT



PACKAGE 1

Balanced Assessment for the Mathematics Curriculum

BERKELEY ■ HARVARD ■ MICHIGAN STATE ■ SHELL CENTRE

Dale Seymour Publications®

Expanded Table of Contents *

Long Tasks	Task Type	Circumstances of Performance
1. Egyptian Statue	30-minute problem, or exercise; applied power in a nonroutine adult context	individual response, or in pairs
2. Books from Andonov	30-minute representation and prediction task; applied power in a nonroutine adult context	individual response, or in pairs
3. Presidential Popularity	30-minute evaluation and recommendation task; applied power in a nonroutine adult context	individual written response
4. Glass Top	30-minute problem; applied power in a nonroutine adult context; open middle	individual response, or in pairs
5. Fermi Estimates	35-minute problem; applied power in a nonroutine adult context; open middle	individual written response
6. Dog Tags	45-minute problem; illustrative application in a nonroutine context from everyday life	individual response, or in pairs
7. Compact-Ness	45-minute definition task; pure mathematics	individual response, or in pairs
8. Blirts and Gorks	30-minute problem; pure mathematics	individual response, or in pairs
9. Keep Taking the Tablets	60-minute investigation; applied power in a nonroutine context from everyday life	individual response, or in pairs

* For explanations of terms that may be unfamiliar, see the Glossary and the *Dimensions of Balance* table in the Introduction.

Advanced High School Package 1

Mathematical Content

Mathematical Processes

Geometry, Space, and Shape: scaling of lengths, areas and volumes of similar figures

transformation with some formulation

Patterns, Functions, and Algebra, with Data: interpret given data; represent as a piecewise-constant graph; infer a function; make predictions

interpretation; representation; formulation

Data, Statistics, and Probability: interpret graphical data; review significance of changes; explain to a newspaper audience

interpretation; manipulation; formulation and explanation

Geometry, Space, and Shape: construction of perpendiculars, and perhaps tangents; Pythagorean theorem

formulation; geometrical construction; explanation

Number and Quantity: estimation of real-life quantities; proportional reasoning and computation

formulation; manipulation; evaluation

Other Mathematics: enumerating the number of combinations of letters under various constraints

formulation; manipulation

Patterns, Functions, and Algebra, with Data and Space: formulate a formal definition from given examples

formulation; manipulation; evaluation

Patterns, Functions, and Algebra: proportional reasoning with linear and quadratic scaling

formulation; transformation

Patterns, Functions, and Algebra, with Data: interpret given data to formulate functions; represent graphically; interpret

interpretation; manipulation; representation; formulation

Expanded Table of Contents

Short Tasks	Task Type	Circumstances of Performance
10. Scale Charts	15-minute problem; illustrative application; some nonroutine mathematical connection	individual response, or in pairs
11. Ford and Ferrari	15-minute problem; illustrative application; nonroutine mathematical connection	individual response, or in pairs
12. Transformation	20-minute exercise; pure mathematics	individual response, or in pairs
13. Triskaidekaphobia	15-minute problem; applied power in a nonroutine adult context	individual response, or in pairs
14. Bathtub Graph	15-minute representation task; applied power in a nonroutine context from everyday life	individual written response
Extended Task	Task Type	Circumstances of Performance
15. Design an Envelope	75-minute design task; applied power in a nonroutine context from everyday life	individual response, or in pairs

Advanced High School Package 1

Mathematical Content	Mathematical Processes
<p>Geometry, Space, and Shape: proportional reasoning; perimeter; scale drawing</p>	<p>transformation of scale; representation in a scale drawing</p>
<p>Patterns, Functions, and Algebra, with Space and Shape: graphical interpretation; reading data from a graph; slope as speed; sketching a map</p>	<p>translation of information and interpretation of a graph of an everyday situation; transformation; representation in a sketch map</p>
<p>Patterns, Functions, and Algebra: transformation of algebraic expressions into given forms</p>	<p>transformation</p>
<p>Other Mathematics: logic and discrete mathematics, using Venn Diagram or other techniques</p>	<p>formulation of a logic problem; manipulation</p>
<p>Patterns, Functions, and Algebra: choosing appropriate variables; sketching a graph</p>	<p>formulation; representation</p>

Mathematical Content	Mathematical Processes
<p>Geometry, Shape, and Space: properties of the triangle, the rhombus, and the rectangle</p>	<p>analysis; manipulation; formulation</p>

Presidential Popularity

Examine the significance of statistical fluctuations.
Write a "letter to the editor" addressing the issue.

Long Task

Task Description

Students are asked to scrutinize a newspaper article's interpretation of some statistical data. The problem calls attention to the notion of margin of error, and asks students to assess whether the statistical fluctuations are significant or insignificant. The student is asked to write a "letter to the editor" in response to the conclusions in the newspaper article.

Assumed Mathematical Background

Students must be comfortable reading data presented in the form of a line graph, but no formal background in statistics is expected.

Core Elements of Performance

- evaluate fluctuations within a set of data, as compared to a stated margin of error
- communicate the idea of margin of error as it applies to this problem

Circumstances

Grouping:	Students complete an individual written response.
Materials:	calculators
Estimated time:	30 minutes

Acknowledgments

The newspaper article, graph, and survey data are from *USA Today* of June 14, 1994, page 10A. Copyright 1994, USA TODAY. Reprinted with permission.



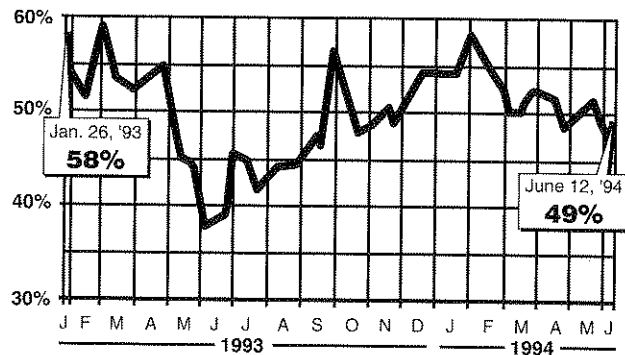
Presidential Popularity

This problem gives you the chance to

- examine the importance of statistical changes reported by a newspaper
- write a "letter to the editor" about mathematical issues

The following information appeared in the June 14, 1994 issue of *USA Today*.

Clinton approval rating up



Source: *USA TODAY*/CNN/Gallup Poll of 756 adults by telephone on June 11–12, 1994.
Margin of error: ± 4 percentage points. By Stephen Conley, *USA TODAY*

The accompanying story, entitled "With Clinton home, voters lighten up," read in part:

With D-Day observances over and President Clinton back home, voters' attitudes toward the president are settling down a bit. Now that attention is back on the economy, health care, and crises in Bosnia and Haiti, a *USA Today*/CNN/Gallup Poll taken over the weekend [of June 12] shows Clinton's job performance rating inching upward to 49% ... It's an improvement from a poll taken [on June 6] as Clinton was in Europe marking the 50th anniversary of the Allied invasion of Normandy, which showed approval dropping to 46% ...



Name

Date

Think about the following issues:

- the meaning of the statement "Margin of error: ± 4 percentage points"
- the change in Clinton's popularity rating, from 46% on June 6 to 49% on June 12, as compared to the margin of error
- the significance or insignificance of this change

Write a letter to the editor of *USA Today* explaining why the assertion "Clinton approval rating up" might be regarded as questionable or misleading. Be concise (editors prefer letters that are brief and to the point) but convincing and give mathematical evidence to support your claims.



A Sample Solution

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Student responses will vary considerably, but a central point should be a comparison between the poll's margin of error ($\pm 4\%$) and the magnitude of the change in popularity between June 6 and June 12 (only 3%). Since the change falls within the margin of error, it should be regarded as insignificant. Therefore, assertions such as "Clinton approval rating up" and "It's an improvement" are not valid conclusions from the poll data.

Students might note that Clinton's popularity could actually have decreased between the two surveys. In view of the margin of error, Clinton's popularity on June 6 could have been 50% instead of the poll's 46%, and his popularity on June 12 could have been 45% instead of 49%. While the poll indicates an increase by 3 percentage points, the President's popularity might have actually decreased by 5 percentage points.

It might also be noted that while this short-term fluctuation was insignificant, historically there have been significant changes in Clinton's popularity. For example, between February 1993 and June 1993, the popularity rating dropped from about 59% to about 37%.

Here is a sample letter to the editor:

Dear Editor,

Your report of June 14 on the popularity of the President is inaccurate. You report that his popularity has gone up since the 6th of June. However, according to your own data, the change in the President's level of popularity is insignificant and should not be reported as an increase.

The margin of error is used to indicate that if another poll were taken at the same time as the original poll, we can be very certain that the difference between the results of the two polls would be less than the margin of error.

In this case the 3% difference between the results of the two polls is less than the margin of error. Even if these two polls were taken at the same time we could have expected this kind of discrepancy between them. Therefore, we can be pretty sure that the difference between the polls is due to the nature of random sampling and not to changes in the average American's opinion of the President.

Sincerely,
Mary Q. Public

Using this Task

Initially, small-group conversations about the problem may be valuable, but students should be left with enough time to think individually and compose an effective letter in response to the task.

Extensions

Students could investigate how the margin of error is determined, based on the size of the sample population.

Characterizing Performance

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This section offers a characterization of student responses and provides indications of the ways in which the students were successful or unsuccessful in engaging with and completing the task. The descriptions are keyed to the *Core Elements of Performance*. Our global descriptions of student work range from “The student needs significant instruction” to “The student’s work meets the essential demands of the task.” Samples of student work that exemplify these descriptions of performance are included below, accompanied by commentary on central aspects of each student’s response. These sample responses are *representative*; they may not mirror the global description of performance in all respects, being weaker in some and stronger in others.

The characterization of student responses for this task is based on these *Core Elements of Performance*:

1. Evaluate fluctuations within a set of data, as compared to a stated margin of error.
2. Communicate the idea of margin of error as it applies to this problem.

Descriptions of Student Work

The student needs significant instruction.

The student exhibits little understanding of the notion of margin of error.

Student A

The explanation offered by this student relies solely on a visual examination of a portion of the graph, and makes no reference to the margin of error. Furthermore, the student compares June ‘93 to June ‘94, whereas the task called for an analysis of June 6, 1994 and June 12, 1994.

The student needs some instruction.

The student compares the change in popularity with the margin of error, but does not interpret the results.

Student B

This student makes numerical comparisons that are somewhat confused, as are the expressions of the inferred conclusions. The interesting point of the composition of the sample is raised in this piece of student work, but the point is made in support of a comparison that is unrelated to the task.

The student's work needs to be revised.

The student numerically compares the changes in popularity with the margin or error and raises pertinent issues, but no complete conclusion is drawn. The work is presented in the requested format of a letter to the editor, but it would benefit from clarification and expansion.

Student C

This student has an understanding of what margin of error implies for this task, but he has not specified those implications numerically.

The student's work meets the essential demands of the task.

The student assesses the significance or insignificance of the change in the popularity rating in view of the given data. The effectiveness of the letter is strengthened by either commenting on the accuracy of the headline, or discussing the significance and insignificance of changes in terms of the historical data shown in the graph.

Student D

This student's response, although terse, gets at the essential elements of the task and evaluates the fluctuations in the data appropriately.

Presidential Popularity ■ Student Work

Student A

Think about the following issues:

- the meaning of the statement "Margin of error: ± 4 percentage points"
- the change in Clinton's popularity rating, from 46% on June 6 to 49% on June 12, as compared to the margin of error
- the significance or insignificance of this change

Write a letter to the editor of *USA Today* explaining why the assertion "Clinton approval rating up" might be regarded as questionable or misleading. Be concise (editors prefer letters that are brief and to the point) but convincing and give mathematical evidence to support your claims.

USA TODAY

To Whom it may concern,

I recently read the article and poll you printed entitled "Clinton's approval rating up". I have found the observation and graph to be correct. Clinton's approval rating even though it appears to have fallen it has actually risen from June of '93 to June of '94. If you read the graph straight from beginning to end it appears that the poll is suggesting that his ratings have fallen, but if you read the graph carefully from June '93 to June '94 you can see that it has actually risen.

Presidential Popularity ■ Student Work

Student B

Think about the following issues:

- the meaning of the statement "Margin of error: ± 4 percentage points"
- the change in Clinton's popularity rating, from 46% on June 6 to 49% on June 12, as compared to the margin of error
- the significance or insignificance of this change

Write a letter to the editor of *USA Today* explaining why the assertion "Clinton approval rating up" might be regarded as questionable or misleading. Be concise (editors prefer letters that are brief and to the point) but convincing and give mathematical evidence to support your claims.

Dear Editor:

In my opinion, the assertion "Clinton approval rating up" might be regarded as questionable or misleading. First of all, ± 4 percentage points margin of error is the very first big mistake. If there is $+4$ error on Jan 26, and -4 error on June 12, 94. The difference of the rating will be $62\% - 45\% = 17\%$. Other word, if there is -4 error on Jan 26, and $+4$ error on June 12, the difference of the rating will be $54\% - 53\% = 1\%$. So, why wasted that much deal of time out of it?

Second of all, ~~or~~ ~~are~~ ~~is~~ did you use the same group of people on Jan 26 and June 12? If not how could you compare their changes of opinion? You may find people who love Clinton for the first ^{time}, but people who against him for the second time. So the rating must go down. But if that was other way around the result will be different totally.

Presidential Popularity ■ Student Work

Student C

Think about the following issues:

- the meaning of the statement "Margin of error: ± 4 percentage points"
- the change in Clinton's popularity rating, from 46% on June 6 to 49% on June 12, as compared to the margin of error
- the significance or insignificance of this change

Write a letter to the editor of *USA Today* explaining why the assertion "Clinton approval rating up" might be regarded as questionable or misleading. Be concise (editors prefer letters that are brief and to the point) but convincing and give mathematical evidence to support your claims. To the Editor

I believe that your USA Today Poll on Clinton's Approval Rating is very Questionable. On your poll it shows that there is a difference of only 3% on both results. 49% Approval on June 12 \pm 46% Approval on June 6, but then there is an error margin of $\pm 4\%$. What this really means is that there is a possibility that there was no change in this poll due to the margin of error. So why right an article on something that technically hasn't changed?

Student D

Think about the following issues:

- the meaning of the statement "Margin of error: ± 4 percentage points"
- the change in Clinton's popularity rating, from 46% on June 6 to 49% on June 12, as compared to the margin of error
- the significance or insignificance of this change

Write a letter to the editor of *USA Today* explaining why the assertion "Clinton approval rating up" might be regarded as questionable or misleading. Be concise (editors prefer letters that are brief and to the point) but convincing and give mathematical evidence to support your claims.

To the editor:

In any poll, the assertion that a popularity rating has gone up or down should not be made. For instance, on June 6, 1994, the polled approval rating for President Clinton was 46%.

On June 12, 1994, his approval rating went up to 49%. However, this poll has a $\pm 4\%$ margin of error. Therefore, the actual approval rating for President Clinton on June 6 could have been 50% ^{or 40%}. If the rating on June 12 was real, there may not have been an actual change in rating, or it could have been higher or lower.

Sincerely,